Wittgenstein:
Teaching and Learning With Turing

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1. Wittgenstein and Turing were mutually “alien” to one another: on opposite sides of a dichotomy between methods of ordinary language and methods of formal logic (Monk).

2. In *Philosophical Investigations* Wittgenstein is concerned to reject Turing’s machine model as an analysis of logic.

3. Turing himself was a computationalist reductionist, i.e., mechanistic functionalist about the mind.
Main Claims

• Wittgenstein and Turing shared a matrix of foundational ideas about the nature of logic.
• They also discussed the nature, limits, and foundations of logic over many years.
• They drew from one another, as they both recognized.
• We have here a confluence of ideas forged over many years, not a conflict.

Wittgenstein $\rightarrow$ Turing
Wittgenstein $\leftarrow$ Turing
Wittgenstein and Turing

1. Wittgenstein influenced Turing to take up an “anthropological” perspective on the foundations of logic, beginning in Turing’s undergraduate years (1932-33) and in discussions stretching into Turing’s time at Bletchley (1944).

2. Turing’s analysis of logic stimulated Wittgenstein to devise his mature approach to the notion of “simplicity”, embodied in the modernist, first-person articulated, interlocutory style of *Philosophical Investigations*. The human form of life in conversation is *fundamental* to logic, philosophy, and life.
Wittgenstein → Turing

Without his study of philosophy with Wittgenstein (specifically, *The Blue and Brown Books*) Turing would not have designed the “Turing Machine”, the idea of the stored program computer, or pursued his further work in the particular way he did. The “Turing Machine”, and much of Turing’s work, has an *anthropological* cast and style. This may be seen by examining

- The Cambridge context of his work with Wittgenstein.
- The argumentation in “On computable numbers, with an application to the *Entscheidungsproblem* (1936).
- Turing’s subsequent writings 1937-1954.
Wittgenstein’s *Philosophical Investigations* was composed under the influence of Turing’s “On Computable Numbers” (sent to him by Turing in early 1937). This paper sparked him to develop his mature, interlocutory style of writing in which the first person is embedded throughout in method.

- He gives up on the ideal of a “gap free” presentation of the nature of logic and develops the idea of rule-following as embedded in everyday phraseology.
- The idea of a partial, rather than a total function is taken to be basic.
- Wittgenstein eliminates the notion of “culture” *Kultur* from the manuscript in 1937, substituting for it the notion of a “form of life” [*Lebensform*], having both biological and ethnological inflections.
Turing Re-Read

• Turing’s philosophical attitude has been distorted by controversies in recent philosophy of mind (Putnam): computationalist and behaviorist reductionisms, functionalism, AI, and the “the singularity”, in which machines will inevitably become the primary drivers of cultural change and creativity.

• Turing was neither a behaviorist nor a reductive mental mechanist. Philosophy of logic, not philosophy of mind, was central for his work on foundations: the human touch, and the ideals of simplicity and shareability.

• Turing emphasized Wittgenstein’s idea of the centrality of everyday language, and the human, end-user “parochialism” it reflects. This parochialism is ubiquitous.
A Different View of Turing

1. Turing had a much more deeply philosophical and far-reaching understanding of the foundations of logic, mathematics, and technology. He focussed on taking what we say and do with words seriously, and on the limits of formal methods and mechanisms, not only their power. Logic and conversation, indirect discourse and anthropology – and not psychology – are primary in all of his work: We offload routines in order to interface.

2. Everyday language, including our “typings” of objects as they occur naturally in science and everyday life, are the framework: Turing stressed human conversation, “phraseology”, “common sense”, and what he called our “cultural search”, the evolving “human community as a whole”, as foundational.
1932-33 Wittgenstein’s “Philosophy for Mathematicians”:
“What counts in mathematics is what is written down: if a mathematician exhibits a piece of reasoning one does not inquire about a psychological process”.

March 1933 Turing read Russell’s *Introduction to Mathematical Philosophy* (Wittgenstein: logic is tautologous).

December 1933 Turing gives a talk to the Moral Sciences Club talk: “the purely logistic view of mathematics is inadequate; mathematics has a variety of interpretations, not just one”.

1933-34, 1934-5 Wittgenstein dictated *The Blue Book* and *The Brown Book* to his (mathematics) students. Turing probably saw lectures, dictations.
Turing’s Analysis of Logic 1935-6

What Turing offered in “On Computable Numbers” was an analysis of our idea of a “step” in a formal system (a “formal operation”) by embedding the idea of “calculation-in-a-logic” in a shared human world: an analogical simplification.

This analysis does not turn on a theory of mental states, mathematics, or logic.

Turing takes the everyday ideas of a “command” and a “calculation” as basic elements of logic and works out a (mathematically robust) “comparison” between the activities of a human and that of a machine.

Like Wittgenstein, Turing takes the *human* notion of calculation as basic or simple, and builds his analogy with machines from there.
The Entscheidungsproblem

As an application of his analysis, Turing resolves the decision problem in the negative:

There is no definite method [algorithm] that can determine, for every statement of mathematics expressed formally in an axiomatic system (using first-order logic), whether or not that statement can be deduced from the axioms.
The problem: What is a “Definite Method”? 

What is

- a formal system?
- a “definite method”?
- a “formal step”?
- an algorithm?
- a logic?
Wittgenstein's and Turing's point

To say what a formal system *is*, we cannot just write down another formal system.

We have to get at this conceptually, philosophically.
Turing’s (1936) Analysis

Turing analyzed what a step in a formal system is by thinking through what it is for, i.e., what is done with it.

The comprehensiveness of his treatment—its lack of “morals”—lies here.

Turing made the very idea of “the simplest steps” in a formal system plain, or “homespun”.

His analysis is simpler, more vivid and satisfactory, than those offered by Gödel and Kleene.
In the end, one must draw in the notion of a human calculator. This requires, not a psychological account, but a logical one: the idea of a shareable human calculating procedure that may be offloaded.

Turing took up a “form of life” or “language-game” stance, not an ideological or metaphysical perspective: he de-psychologized the notion of “logic”.
Turing 1936: Eliminating “states of mind”

We avoid introducing the notion of a ‘state of mind’ by considering a more physical and definite counterpart: it is always possible for the computor to break off from his work, to go away and forget all about it, and later to come back and go on with it. If he does this he must leave a note of instructions (written in standard form) explaining how the work is to be continued. This note is the counterpart of the ‘state of mind’.
Turing (1936) on Simplicity

- A human computor works locally, step-by-step, and can only take in a certain number of symbols at a glance.
- The computor takes in “simple operations ... so elementary that it is not easy to imagine them further divided”.

But this is not a psychological point, it is rather a logico-mathematical and *philosophical* one: commands should be easy to take in.
Turing 1936: Absoluteness

Turing’s analysis of a “step” in a formal system is altogether independent of *which* formal system we are speaking of, or *which* “states of mind” are actually used.

This is why his model demands that we embed command routines in “forms of life”, with techniques.

(Wittgenstein: forms of life, not culture.)
Turing 1936: Absoluteness

Turing crafts his argument in “On Computable Numbers” so that even an intuitionistic logician who rejects the law of the excluded middle in infinite contexts can accept his analysis of the idea of a “step” in a formal system.

It is not part of our notion of “following a rule step-by-step” that we do or do not obey the law of excluded middle, that the answer must always be Yes or No.
Turing 1936 Diagonalizing: *partial* functions are basic

\[
\begin{array}{cccccc}
\downarrow & 1 & 1 & 0 & \downarrow & \ldots \\
1 & 0 & 0 & 0 & 1 & \ldots \\
0 & 1 & \downarrow & 0 & 0 & \ldots \\
1 & 1 & 0 & \downarrow & 0 & \ldots \\
1 & 1 & 1 & 1 & 1 & \ldots \\
\ldots & & & & & \\
\ldots & & & & & \\
\ldots & & & & &
\end{array}
\]

Therefore one cannot diagonalize out of the enumeration of Turing Machines.
The Stored Program Computer Concept

• This is the idea of a machine that can work on its own commands (Wittgenstein: the idea of a machine “symbolizing its own actions”)

• Turing’s Universal Machine makes possible the ubiquity of computational processing in our world, and its indefinite extent of application and ability to compress what is definable.

• The Universal Machine shows that there are no dichotomies between hardware, data, and software

• The Universal Machine shows us the robustness in the idea of the fluidity of “simplicity”
Turing’s Proof (1936)

Turing used a *tautological* construction to show the limits of (formalized) logic from the *inside*, without a theory of logic or “states of mind”. His proof does *not* produce a contradiction, or even use negation.

**Do What You Do**

The point of this is purely philosophical, not only mathematical. The human interface, the human context of a shareable command, is *demonstrated* to be fundamental to the nature of logic.
Wittgenstein, RPP I §1096 (MS 135, 1947)

Turing’s “Machines”. These machines are humans who calculate. And one might express what he says also in the form of games. And the interesting games would be such as brought one via certain rules to nonsensical instructions (unsinnigen Anweisungen)... One has received the order “Go on in the same way” when this makes no sense, say because one has got into a circle. For that order makes sense only in certain positions. (Watson.)
What is a Turing Machine?

Blythe House London
What is a Turing Machine?
The statement of the type principle given below was suggested by lectures of Wittgenstein, but its shortcomings should not be laid at his door.
Turing, “The Reform of Mathematical Notation” (1942-44)

We should conduct an extensive examination of current mathematical, physical and engineering books and papers with a view toward listing all commonly used forms of notation and examine them to see what they really mean. This will usually involve statements of various implicit understandings as between writer and reader. But the laying down of a code of minimum requirements for possible notations should be exceedingly mild, avoiding the straightjacket of a logical notation.
Turing, “The Reform of Mathematical Notation” (1942-44)

It would not be advisable to let the reform [of notation] take the form of a cast-iron logical system into which all the mathematics of the future are to be expressed. No democratic mathematical community would stand for such an idea, nor would it be desirable.
Even while at Bletchley, Turing worked at the foundations of logic. He explored the history of notations, drawing comparisons as to their usefulness, running through Leibniz, Boole, Peano, etc.
But what about Turing and AI?
Turing, “Intelligent Machinery” (1948)

A great positive reason for believing in the possibility of making thinking machinery is the fact that it is possible to make machinery to imitate any small part of a man. That the microphone does this for the ear, and the television camera for the eye, are commonplaces. One can also produce remote controlled robots whose limbs balance the body with the aid of servo-mechanisms. Here we are chiefly interested in the nervous system. We could produce fairly accurate electrical models to copy the behaviour of nerves, but there seems very little point in doing so. It would be rather like putting a lot of work into cars which walked on legs instead of continuing to use wheels.
Turing, “Intelligent Machinery”

One way of setting about our task of building a “thinking machine” would be to take a man as a whole and to try to replace all the parts of him by machinery. He would include television cameras, microphones, loudspeakers, wheels and ‘handling servo-mechanisms’ as well as some sort of ‘electronic brain’. This would of course be a tremendous undertaking. The object if produced by present techniques would be of immense size, even if the ‘brain’ part were stationary and controlled the body from a distance.
Turing, “Intelligent Machinery”

In order that the machine should have a chance of working things out for itself it should be allowed to roam the countryside, and the danger to the ordinary citizen would be serious. Moreover even when the facilities mentioned above were provided, the creature would still have no contact with food, sex, sport and many other things of interest to the human being. Thus although this method is probably the “sure” way of producing a thinking machine it seems to be altogether too slow and impracticable.
Turing, “Intelligent Machinery” (1948): The “Intellectual” Search

We might arrange to take all possible arrangements of choices in order, and go on until the machine proved a theorem which, by its form, could be verified to give a solution of the problem ... Further research into intelligence of machinery will probably be very greatly concerned with “searches” of this kind. We may ... call such searches “intellectual searches”.
It may be of interest to mention two other kinds of search in this connection. There is the genetical or evolutionary search by which a combination of genes is looked for, the criterion being survival value. The remarkable success of this search confirms to some extent the idea that intellectual activity consists mainly of various kinds of search.
Turing, “Intelligent Machinery”, The Cultural Search

The remaining form of search is what I should like to call the ‘cultural search‘... [T]he isolated man does not develop any intellectual power. It is necessary for him to be immersed in an environment of other men, whose techniques he absorbs during the first 20 years of his life. He may then perhaps do a little research of his own and make a very few discoveries which are passed on to other men. From this point of view the search for new techniques must be regarded as carried out by the human community as a whole, rather than by individuals.
Main Lessons of The Wittgenstein-Turing Foundational Perspective

• The context of an action or algorithm within our social world makes it what it is.

• “Intelligence”, “Thinking” and “Culture” consist in a variety of different kinds of human search.

• Algorithms do what they do, but only against a human backdrop of understanding.

• Algorithms are not, in general, neutral objects, but involve qualitative aspects: biases, values, a point; they are parochial and powerful elements of larger systems.

• What we do with and say about algorithms makes them what they are: they and we are accountable to one another.
Work and Leisure: Wittgenstein to Turing 1939

You smoke cigarettes every now and then and work. But if you said your work was smoking cigarettes, the whole picture would be different.

Idea: What used to be thought of as leisure, “gas” or “jaw” or “nonsense” in the foundations of mathematics (everyday discussion among people, heuristics and models to help people see the point of an abstraction, visualizations, etc.) has become absolutely fundamental to what logic and mathematics are.
Our Social Nature: Acknowledgment (Cavell)

- The problem of other minds is more fundamental than that of the ‘external’ world.
- We want to be freely recognized by others who are also free and worthy of being recognized by.
- We want to be known, become someone, to have our say.
- Rousseau: *Amour Propre* and passionate utterance.

This seems to be what is driving our uses of mobile technology and especially social media: ethics and voice in ordinary life, who speaks on behalf of whom, is fundamental.
Philosophers and the humanities need a voice at the table in our world of emerging media!
Mellon Sawyer Seminar, Boston University, 2016-2018
Humanities and Technologies at the Crossroads,
Where Do We Go From Here?

www.mellonphilemerge.com
Journalism and the Search for Truth in an Age of Social Media

Livestreamed. Held between the French runoff to the Presidential Election and the final French Election of Macron. Historians, sociologists, philosophers and journalists weighed in on “fake” news and what to do about it.
Philosophical Platforms: Arendt, Nietzsche, Wittgenstein

How to preserve the diversity and relevance of interpretations in a world of automatic manufacture of semantic ontologies, fan fiction, and elaborated academic platforms of collaboration. Philosophical discussion of the nature of “documents”, “texts”, and “integrity” of a philosophical corpus.
Philosophy of Popular Culture: Scepticism, Care, and Ordinary Life, 10/23/17

Sandra Laugier, Professor of Philosophy, Université de Paris 1, Panthéon-Sorbonne. Scepticism, informal representation, and ordinary life in and out of cinema and television.
Is this an Epistemological Revolution?  
Big Data and the Philosophy of Science, 11/27/17

Sabina Leonelli (University of Exeter) and John Symons (University of Kansas) on data inequity, management of biological data sets, and epistemological challenges of software error. Deep AI and “Post Human Science”.


Turing’s Imitation Game:
What does it tell us about the Nature of Intelligence? 12/11/2017

Professors Jack Copeland and Diane Proudfoot (University of Canterbury, New Zealand, co-founders and co-directors of the Turing Archive for the History of Computing and of the Turing Centre at the Swiss Federal Institute of Technology, Zürich). Intelligence as an “emotional” concept. Media and AI representations of “the electronic brain” from the 1950s show “fake news” in action.
Digitizing Human Rights, Archiving Activism, 1/22/2018

A discussion of several projects promoting ethics and activism with technology as well as archiving of activism. Speakers include Alexa Koenig (UC Berkeley Human Rights Center), Anat Biletzki (B’Tselem), Peter Manning (Sociology, Northeastern) and members of the Charlie Hebdo Archives at Harvard University.
Day of Apparatgeist, 2/5/2018

The social and epideictic as primary. Empirical explorations of how people see and feel about one another in the context of mobile technology. How human uses of mobile technology were predicted by this sociological theory developed by Katz and Aakhus in 1999.
Open Access and Research into History: Issues of Copyright, 2/20/2018

Casey Westerman (Archivist for the Institute for Advanced Study) on Gödel’s MaxPhil notebooks and the complex issues around their transcription and editing. Peter Suber (Director of the Harvard Office for Scholarly Communication, Senior Researcher at the Berkman Klein Center for Internet and Society) commented.
Accountability in an Age of Algorithms: How Should Ethics and Technology Converse?

Susan Schneider (Philosophy, Cognitive Science University of Connecticut; IAS Princeton, Yhouse, NY Ethics and Technology Group, Yale Interdisciplinary Center for Bioethics). Triple roundtable of faculty from the Boston area, 3/15-16/18.

1. Algorithms and the Criminal Adjudicative Process
2. Privacy
3. AI and the Internet of Things
Human Plasticity and Human-Machine Interface
4/9/2018

Anders Sandberg (Oxford), Nicholas Agar (Victoria, Wellington University), Joseph A. Stramondo (San Diego State), Serife Teki (Boğaziçi University).
Social scientists, philosophers, policy-makers, and computer scientists explore the social, behavioral, and psychological dimensions of this new technological terrain.
Grappling with the Futures: Insights from Philosophy, History and Science, Technology and Society, 4/29-30/18

Futures Studies: a variety of methods for predicting, forecasting, anticipating, controlling, imagining, and shaping multiple futures and levels of expectation among humans. Philosophers, historians, and science, technology and society (STS) scholars.
Jon Burmeister, PhD (Boston College, Philosophy PhD, 2011; College of Mount St. Vincent)

Hegel’s notion of living logic and the evolution of language; Arendt on work and leisure, freedom, politics, human nature. Nineteenth century continental philosophy and philosophy of technology. Recipient of a 2016-2017 NEH “‘Enduring Questions’ Grant on the subject of Work and Leisure” (www.workandleisure.org)
Seena Eftekhari (Kansas, Philosophy PhD, 2018, Instructor, Tufts University)

The connection between liberal conceptions of personhood and the status of the capitalist economic liberties. Areas of political philosophy such as liberalism, socialism, distributive justice, freedom, and Austrian political economy.
David Leslie, PhD (Yale, Political Science PhD, 2010) Harvard Social Studies, Princeton Center for Human Values, Permanent Research Fellow, Ethics, Alan Turing Institute, London

Social and ethical consequences of emerging technologies; how the biospherically and geohistorically ramifying scope of scientific innovation puts pressure on conventional categories, norms, and human practices. Anthropogenic extinction risk and intergenerational justice.

Social and political philosophy, with a focus on informal political representation. Philosophy of law, black political thought, ethics, feminism, Africana philosophy and bioethics.
Laura Specker Sullivan, PhD (University of Hawai‘i at Manoa, with a Graduate Certificate in Japanese Studies, 2015; Fellow at the Harvard Center for Bioethics, Harvard Medical School (2017-2018), College of Charleston)

Justification of cross-cultural claims in global bioethics, Japanese medical ethics, human-machine interface, feminism and bioethics in a global context.
Zeynep Soysal (Harvard, PhD 2017, BU Mellon Postdoctoral Fellow 2017-2018, University of Rochester)

Epistemology of set theory and the infinite, history, pedagogy, and philosophy of logic and mathematics. Epistemological gate-keeping and deference to expertise in conditions of incompleteness and potential undecideability, including journalism. Mediator. Active in Turkey in promoting the use of social media to encourage democratic participation.
Anandita Mukherji (Boston University PhD 2018, BU Mellon Teaching Assistant Fellow 2017-2018, Assistant Professor of Philosophy, Regis University, Denver).

Global poverty and limits of theory and practice of redistribution; institutional global justice; the nature of liberty considerations in duties not to harm; Arendt, Marx and political freedom.
Tiffani Lewis-Lockhart (University of Richmond B.A. (Philosophy), enrolled BU MA (Emerging Media), BU School of Social Work)

Continental philosophy, especially social theory, development of diverse educational teams, Emerging Media with a focus on social networks, representation and experiences of African Americans; ethics of voice and normative uses of Emerging Media.
Future Plans

• Pedagogy: Undergraduate, General Education
• Graduate Student, Post-Doc Interdisciplinarity Support
• Research: Experimental Philosophy
• Social Media: Twitter, Connections, Outreach to the Public
• Suggestions, comments welcome!
Bibliography, Juliet Floyd on Turing and Wittgenstein


Thank You!